Objective: 3) Factor out -1 in GCF
4) Factor by grouping.

Factor out the GCF

① $-8y^4 - 12y^2$

↑

When the highest-degree term has a negative coefficient, we should (usually) factor out a -1 in the GCF.

Step 1: Find the GCF $\quad -4y^2$

Step 2: Factor out the GCF

$$= -4y^2\left(\frac{-8y^4}{-4y^2} + \frac{-12y^2}{-4y^2}\right)$$

$$= \boxed{-4y^2(2y^2 + 3)}$$

② Factor $-8y^4 + 12y^2$

Step 1: GCF $= -4y^2$, same as ①

Step 2: $-4y^2\left(\frac{-8y^4}{-4y^2} + \frac{12y^2}{-4y^2}\right)$

$$= \boxed{-4y^2(2y^2 - 3)}$$

Note: If you factor out a -1 in the GCF, all the original signs are changed.

In ①: (−) (−) became (+) (+)

In ② (−) (+) became (+) (−)

If you are prone to sign errors, it's a good idea to check the signs at the beginning and end of your work to see that you changed them all.

Factor by grouping:

- Can be used for other situations, but in Math 45 will only be used when there are four terms
- We separate the terms into two groups of two terms each
- Factor out GCF from each group of two terms.
- Then look to factor out one more GCF.
- You may need to rearrange the terms.

(3) Factor by grouping $\underbrace{3x-3y}_{\text{group \#1}}+\underbrace{ax-ay}_{\text{group \#2}}$.

Step 1: Identify two groups with 2 terms in each group.

Step 2: Factor out GCF from each group.

$$3(x-y)+a(x-y)$$

Step 3: Check that a 3rd GCF exists.

Step 4: Factor out the GCF

$$\boxed{(x-y)(3+a)}$$

GCF | left from 1st term | left from 2nd term

Note 1: If the first term in either group has a negative coefficient, factor out -1 in GCF

Note 2: If the terms in each group are not in standard form (descending order by exponent) you may have to rearrange & start again.

Note 3: If there's no GCF at step 3,

- check your signs. eg $(x+y)$ and $(x-y)$ might be a sign error
- check your arithmetic eg $(x+y)$ and $(2x+y)$ might be an arithmetic error
- make sure the problem was in standard form.
- Re-group if necessary — some groups just don't work.

Factor by grouping:

(4) $5x - 5y - 4bx + 4by$

$= 5(x-y) - 4b(x-y)$

$= \boxed{(x-y)(5-4b)}$

GCF; left 1st term; left 2nd term

Note: must take out the -1 with GCF for 2nd group. changes (−) (+) to (+) (−)

(5) $3x^3 + 12x^2 - 6x - 24$

$= 3x^2(x+4) - 6(x+4)$

$= (x+4)(3x^2-6)$ ← Notice GCF 3 in 2nd ~~factor~~!

$= \boxed{3(x+4)(x^2-2)}$ ← Factor out 3. from 2nd factor.

Method 2: Take out GCF = 3 first!

$= 3(x^3+4x^2-2x-8)$ ← uh!-oh, extra parentheses. Let's make them brackets.

$= 3[\underbrace{x^3+4x^2}_{\text{1st group}} - \underbrace{2x-8}_{\text{2nd group}}]$

$= 3[x^2(x+4) - 2(x+4)]$

$= 3(x+4)[x^2-2]$

$= \boxed{3(x+4)(x^2-2)}$

Note: The advantage of taking out the GCF first is that all the #s are smaller after that. The disadvantage is the extra set of brackets/parentheses may look more confusing. Both methods are valid.

Note: Factor by grouping problems can be checked using FOIL

Factor completely.

⑥ $-12z^3 + 16z^2 - 8z$ ← 3 terms, GCF

$= \boxed{-4z(3z^2 - 4z + 2)}$

* ⑦ $2t^4 - t^3 - 2t + 1$ ← 4 terms, grouping.

$= t^3(2t-1) - 1(2t-1)$

$= \boxed{(2t-1)(t^3-1)}$

GCF

Note: In 6.4, we'll learn how to factor t^3-1. But for now all we know is grouping + GCF

⑧ Find the missing factor.

$9 - 2x^{-1} + 8x^{-3} = x^{-3} \cdot ?$

Key: The question mark (?) represents a quantity in parentheses — it will have 3 terms and these will be the leftover parts after we factor out x^{-3}.

Do step 2 only!

$9 - 2x^{-1} + 8x^{-3}$

$= x^{-3}\left(\frac{9}{x^{-3}} + \frac{-2x^{-1}}{x^{-3}} + \frac{8x^{-3}}{x^{-3}}\right)$

$= \boxed{x^{-3}(9x^3 - 2x^2 + 8)}$

subt. exp:
$-1-(-3)$
$-1+3$
2

⑨ Factor the GCF from $\frac{1}{5}w^3 - \frac{2}{25}w$

When the coefficients are fractions, the GCF is a fraction, too. Numerator = GCF of numerators
Denominator = LCD of denominators
Variables = as usual.

GCF numerators	1	-2	$\Rightarrow$	1
LCD denominators	$\frac{1}{5}$	$\frac{-2}{25}$	$\Rightarrow$	$\frac{1}{25}$
Variables	w^3	w	$\Rightarrow$	w

GCF $= \frac{1}{25}w$.

Factor out GCF as usual: (it just involves fractions)

$$= \frac{1}{25}w\left(\frac{\frac{1}{5}w^3}{\frac{1}{25}w} - \frac{\frac{2}{25}w}{\frac{1}{25}w}\right)$$

$$= \boxed{\frac{1}{25}w\left(5w^2 - 2\right)}$$

Note: If you did the LCD part correctly, your fractions should disappear.

Divide fractions

$$\frac{1}{5} \div \frac{1}{25}$$
$$= \frac{1}{5} \cdot \frac{25}{1}$$
$$= 5$$

Divide fractions

$$\frac{2}{25} \div \frac{1}{25}$$
$$= \frac{2}{\cancel{25}} \cdot \frac{\cancel{25}}{1}$$
$$= 2.$$